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VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS

Faculty: CA Arijit Chakraborty

Integration

$$\int f(x)dx$$

- Integration is the **reverse process of differentiation.**
- **Anti-derivative**
- Integration is the **inverse operation of differentiation** and denoted by the symbol : S :
- The symbol is a stylized *S* to indicate summation
- Integral calculus was primarily invented **to determine the area bounded by the curves dividing the entire area into infinite number of infinitesimal small areas** and taking the sum of all these small areas.

What is an Integral ‘s use ?

- Economics / Finance
  - *Profit of a company down the years*
  - *Customer addition*
  - *Savings people make during their life time..*
- Total audit risk

Indefinite and Definite Integrals

Indefinite

$$\int f(x)dx$$

Definite

$$\int_{x_1}^{x_2} f(x)dx$$

Indefinite Integral

- The indefinite integral is a family of functions

$$\int x^3 dx = \frac{1}{4}x^4 + C$$

$$\int \left(3x^{-2} + 4\right) dx = -3x^{-1} + 4x + C$$

- The + C represents an arbitrary constant
  - *The constant of integration*

8.B.2 : Basic formulae

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$$

- The derivative of the indefinite integral is the original function

$$\frac{d}{dx}\int f(x)dx = f(x)$$

Properties of Indefinite Integrals

- The power rule

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$$

- The integral of a sum (difference) is the sum (difference) of the integrals

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

Constant Multiplier Rule

$$\int kf(x) dx = k \int f(x) dx, \qquad k \text{ is a constant}$$

The general solution of integrals of the form kx^n

- From Activity 2 above, the general solution of integrals of the form $\int kx^n dx$, where k and n are constants is given by:

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + c \; ; \text{ where } n \neq -1 \text{ and } c \in \mathbb{R}$$

$$\int x^{-1}\, dx = \int \frac{1}{x}\, dx = \ln|x| + C$$

Indefinite Integrals of Exponential Functions

1. $\int x^n dx = \frac{x^{n+1}}{n+1} (n \neq 1)$

2. $\int \frac{1}{x} dx = \ln |x|$

3. $\int e^x dx = e^x$

4. $\int a^x dx = \frac{a^x}{\ln a}$

- $$\int e^x \, dx = e^x + C$$
- $$\int e^{kx} \, dx = \frac{e^{kx}}{k} + C$$
- $$\int a^x \, dx = \frac{a^x}{\ln a} + C$$
- $$\int a^{kx} \, dx = \frac{a^{k \cdot x}}{k(\ln a)} + C$$

- Q.1: Evaluate the following: $\int 4x^5 - 3x + 3 / x \, dx$
- Solution:
- $\int (4x5 - 3x + 3 / x) dx$
- $4x^6 / 6 - 3x2 / 2 + 3\ln x + C$
- Where C is integral constant.

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c \quad , \quad n \neq -1$$

Standard formula

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} \qquad (for \ n \neq -1)$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b|$$

$$\int \frac{1}{x^2-a^2} dx = \begin{cases} \frac{1}{2a} \ln \frac{a-x}{a+x} & for \ |x| < |a| \\ \frac{1}{2a} \ln \frac{x-a}{x+a} & for \ |x| > |a| \end{cases}$$

8.B.4 :INTEGRATION BY PARTS

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Choose u in this order: **LIATE**

Logs
Inverse
Algebraic
Trig
Exponential

$$\int f'(x)f(x)dx = \frac{1}{2}(f(x))^2 + C$$

$$\int (4x+5)(2x^2+5x)dx = \frac{1}{2}(2x^2+5x)^2 + C$$

8.B.5 METHOD OF PARTIAL FRACTION

- This process of taking a rational expression and decomposing it into simpler rational expressions that we can add or subtract to get the original rational expression is called **partial fraction** decomposition.
- Many **integrals** involving rational expressions can be done if we first do **partial fractions** on the integrand

Analysis

- $\int 3x+11 \, dx / x2-x-6$
- $3x+11/(x-3)(x+2)$
- $=A/ x-3+B /x+2$
- Now we need to choose AA and BB so that the numerators of these two are equal for every xx. To do this we'll need to set the numerators equal.

- $3x+11=A(x+2)+B(x-3)$
- So $A+B = 3$
- And $2A - 3B = 11$
- So $B = --1$, $A = 4$
- Hence ,
- $\int 3x+11/ x2-x-6\, dx$
- $=\int 4/(x-3)-1/(x+2)dx$
- $=\int 4/ (x-3)\, dx-\int 1/(x+2)dx$
- $=4\ln|x-3|-\ln|x+2|+c$

Pg 8.32 Example

- **Example :** Find the equation of the curve where slope at (x, y) is 9x and which passes through the origin.
- **Solution:**
- Dy/ dx = 9x
- \òdy = or y = $9x^2/2 + c$
- Since it passes through the origin, c = 0; thus required eqn . is $9x^2 = 2y$.

8.B.6 DEFINITE INTEGRATION

- Suppose F(x) dx = *f*(x)
- As x changes from a to b the value of the integral changes from *f* (a) to *f* (b). This is as

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

- ‘b’ is called the upper limit and ‘a’ the lower limit of integration.

Tabulation of Integrals

$$F(x) = \int f(x)dx$$

$$I = \int_a^b f(x)dx$$

$$I = F(x)\Big]_a^b = F(b) - F(a)$$

Definite integral : Properties

Sum/Difference: $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

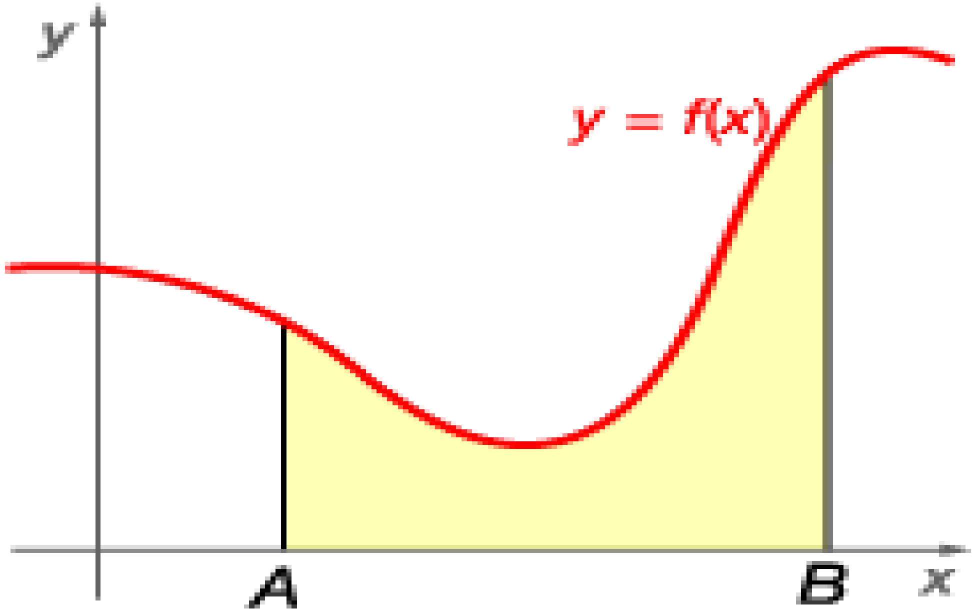
Constant multiple: $\int_a^b k \cdot f(x)dx = k \int_a^b f(x)dx$

Reverse interval: $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Zero-length interval: $\int_a^a f(x)dx = 0$

Adding intervals: $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

Area Under the Curve



How do we find areas under a curve,
but above the x-axis?

Worked examples- The definite integral

3.	$\displaystyle\int_0^1 \mathbf{2}dx$	Let I $= \int_0^1 2dx$ $= [2x]_0^1$ $= [2(1) - 2(0)]$ $\therefore$ **I** $= 2$

THANK YOU